

A Missing Piece  
in an Elementary School Mathematics Teacher's  
Knowledge Base

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How do we judge whether an elementary school teacher is good at teaching mathematics? Most people responding to this question would agree that it is how well the teacher's students learn the mathematics. Elementary school teachers learn to teach mathematics in some type of teacher education program. In California and several other states, these teacher education programs are two-part programs. The first part consists of the undergraduate degree program where prospective teachers develop their content knowledge base. The second part is the post-baccalaureate credential program where they develop their pedagogical knowledge base.

What aspect of the knowledge base that prospective teachers develop in their teacher education programs makes them a successful mathematics teacher? Several people have tried to answer this question over the years. Some have focused on the mathematical content knowledge of the elementary teacher (Ma, 1999; Ball, 1990). Others have examined elementary teachers pedagogical preparation (Darling-Hammond, 2000; Grossman, et. al., 2001). Despite the abundance of research on both sides, no one can yet make any reliable generalizations (Wilson, Floden, & Ferrini-Mundy, 2001). To study this question, we first need to understand elementary school mathematics teaching.

What Is Elementary School Mathematics Teaching?

Shulman (1987) described teaching in general as follows:

A teacher knows something not understood by others, presumably the students. The teacher can transform understanding, performance skills, or desired attitudes or values into pedagogical representations and actions. These are ways of talking, showing, enacting, or otherwise representing ideas so that the unknowing can come to know, those without understanding can comprehend and discern, and the unskilled can become adept. Thus, teaching necessarily begins with a teacher's understanding of what is to be learned and how it is to be taught. It proceeds through a series of activities during which the students are provided specific instruction and opportunities for learning, though the learning itself ultimately remains the responsibility of the students. Teaching ends with new comprehension by both the teacher and the student. (p. 7)

Elementary school mathematics teaching begins with the teacher's understanding of the mathematical content to be taught. The teacher knows how to "do" the mathematics, i. e., understands the concepts and truths concerning that mathematics. The teacher then decides how to present the mathematics. Based on the students' background in mathematics, the teacher selects the explanations and instructional strategies that will help the students build on their current background and develop a thorough understanding. After selecting the strategies, the teacher designs and implements a series of activities aimed at facilitating all students' learning of the mathematics. These activities may include direct instruction, group work, guided discovery, or any other of the many pedagogical tools available. The teacher then assesses the students' level of understanding, diagnosing areas of misunderstanding, and designs additional activities to alleviate those misunderstandings.

Based on this description of elementary school mathematics teaching, an elementary school teacher certainly needs a strong conceptual understanding of mathematical content and an equally thorough understanding of pedagogical skills and practices. However, the lack of any research conclusively connecting either of these two sets of knowledge with student success in learning mathematics seems to indicate that we are missing something.

### The Three Types of Knowledge Required for Teaching Elementary School Mathematics

Wilson, Floden, and Ferrini-Mundy (2001), in their extensive review of teacher preparation research, pointed out that there may be another aspect of teacher knowledge, in addition to content and pedagogical knowledge, which may help explain the non-conclusive findings of

research. Ball (2000) called it “a special amalgam of knowledge that links content and pedagogy,” while Stengel (1997) called it “knowledge for teaching.” This third aspect of a teacher’s knowledge was discussed extensively by Shulman (1986). He called it “pedagogical content knowledge.” The National Council for Accreditation of Teacher Education (NCATE) recently recognized pedagogical content knowledge as a distinct and important aspect of a teacher’s knowledge base. In the 2002 edition of its *Professional Standards for the Accreditation of Schools, Colleges and Departments of Education*, NCATE discusses three types of Candidate Knowledge, Skills and Dispositions (Standard 1). They are (1) Content Knowledge for Teacher Candidates, (2) Pedagogical Content Knowledge for Teacher Candidates, and (3) Professional and Pedagogical Knowledge and Skills for Teacher Candidates. NCATE will begin evaluating teacher preparation programs on the strength of their candidates’ pedagogical content knowledge, in addition to their content knowledge and pedagogical skills, in 2002. The purpose of this article is to begin to explore pedagogical content knowledge in the context of elementary school mathematics, and its relationship to students’ learning of mathematics.

### What is Mathematical Pedagogical Content Knowledge?

Shulman (1986) described pedagogical content knowledge in general as follows:

Within the category of pedagogical content knowledge I include, for the most regularly taught topics in one’s subject area, the most useful forms of representations of those ideas, the most powerful analogies, illustrations, examples, explanations, and demonstrations—in a word, the ways of representing and formulating the subject that make it comprehensible to others... Pedagogical content knowledge also includes an understanding of what makes the learning of specific topics easy or difficult: the concepts and preconceptions that students of different ages and backgrounds bring with them to the learning of those topics. If those preconceptions are misconceptions, which they so often are, teachers need knowledge of the strategies most likely to be fruitful in reorganizing the understanding of learners, because those learners are unlikely to appear before them as blank slates. (p 9)

Mathematical pedagogical content knowledge is the knowledge it takes to teach a particular mathematical topic in a way that the topic and the reasoning surrounding it makes sense to a particular learner or a whole class, based on what they currently know or do not know (Ball, 2000). Teachers who have mathematical pedagogical content knowledge are able to break the mathematics down into components that their

students can understand and link together. These teachers understand the mathematical concepts that form the foundation of the topic being taught and how those concepts interact. They have the kind of understanding that enables them to provide multiple explanations and use multiple strategies so that all students can learn (NCATE, 2000).

Mathematical pedagogical content knowledge is the ability to analyze students' background knowledge and beliefs on a topic, and then, based on that analysis, organize concepts and components in the way students need in order to comprehend the topic. It is the knowledge of which components traditionally cause difficulty for student learning. It is the knowledge to select the most useful activities and explanations that will address the difficulties, help the students correct any misconceptions, and build upon the students' knowledge base to help them learn the desired components and interactions and thus comprehend the mathematics.

Teachers with mathematical pedagogical content knowledge understand their students' knowledge and belief structures, including their inconsistencies and misconceptions, and select and plan activities in a way that will help students correct the misunderstandings and connect the new learning to the students' existing knowledge base. Teachers with mathematical pedagogical content knowledge know which components of a topic are most likely to be difficult for students to assimilate and why. Teachers with mathematical pedagogical content knowledge also know a variety of ways of explaining and demonstrating different topics, their concepts and their interactions. Based on this knowledge, they choose the most appropriate activities to help students comprehend the concepts and their interactions, paying particular attentions to those areas that normally cause difficulty for students. Vignette #1 shows how mathematical pedagogical content knowledge influenced a teacher's decision on what instructional materials and activities to use.

*Vignette #1:*

*Addition Algorithm — Selecting the Most Appropriate Activity*

As a first grade teacher, Melissa was ready to start introducing her students to the addition algorithm. Her students were well on their way to mastering their addition basic facts. Melissa knew that the addition algorithm is based on place value, and that a good understanding of place value is necessary for students to understand the algorithm. To build on her students' existing knowledge, Melissa planned activities to introduce the addition algorithm using place value manipulatives. She gave her students the problem  $27 + 36$ . She had them make each number

using the place value manipulatives (2 tens and 7 ones; 3 tens and 6 ones). She had them add (combine) the ones first (13 ones). Since this was bigger than 10, they traded 10 ones for 1 ten, giving them 1 ten and 3 ones (in place of 13 ones). They then combined their tens to get 6 tens. Thus, their answer was 6 tens and 3 ones or 63. They continued working with the manipulatives, and describing what they were doing until Melissa was sure they understood how the algorithm worked.

Vignette #2 shows how mathematical pedagogical content knowledge helped a teacher analyze a student's difficulties and employ an effective activity to help the student overcome the difficulties and understand the subtraction algorithm.

*Vignette #2:*

*Subtraction Algorithm — Knowing the Source of Misunderstanding*

Samantha was substitute teacher in a fifth grade classroom. The lesson plan left by the teacher was on reviewing the subtraction algorithm. Samantha was to pass out the subtraction review worksheet and help the students who were having trouble with the problems. As Samantha was walking around the classroom, she notice that Sarah was crying. "What is wrong?" she asked. Sarah just shook her head. "Sarah can't subtract!" another student piped in. Samantha knew that one of the biggest problems students have in understanding the subtraction algorithm was the idea of "regrouping." It was time for the morning break, and Samantha asked Sarah to stay in the classroom with her and they would work on the subtraction together. Samantha found some play money in the materials cabinet, and begin showing Sarah how to subtract using the money. She used the problem  $51 - 27$ . She had Sarah make 51 using the money (5 tens and 1 one). Samantha then asked Sarah to take away the 7 ones. "I can't" responded Sarah "there aren't enough." Samantha showed Sarah how to "trade" 1 ten for 10 ones, so that Sarah now had 4 tens and 11 ones. "Now can you subtract the 7 ones?" ask Samantha. Sarah did, and then took away the 2 tens to get an answer of 2 tens and 4 ones or 24. Samantha and Sarah continued practicing with the money until the break was over. At the start of class, Samantha told Sarah she could continue using the money to do the problems on the worksheet if she wanted. As the students continued to

work, Samantha noticed that Sarah was not using the money but trying to do the problems without it, as before. She walked over to remind Sarah it was OK to use the money. “I don’t need it!” responded Sarah, “I understand how to subtract now!” Because of Samantha’s mathematical pedagogical content knowledge, she knew that not understanding regrouping in subtraction was a likely reason that Sarah was having trouble. She used an activity which turned the act of regrouping into the concrete action of trading 1 ten for 10 ones. This, in fact, was Sarah’s problem, and once Sarah understood the idea of regrouping, she could successfully “do” the algorithm.

How is this knowledge different from elementary school mathematics content knowledge? How is it different from pedagogical skills and knowledge? Many readers who specialize in elementary school pedagogy may believe that mathematical pedagogical content knowledge is a form of mathematics content. Many elementary school mathematics content specialists will read this and conclude that it is certainly pedagogical knowledge. Both will be right in their conclusions and beliefs. Mathematical pedagogical content knowledge is the *interaction* of mathematics content and pedagogy (Ball, 2000). Knowing how to break a mathematical topic down into its structural components, and knowing how those components interact is certainly mathematical content. The ability to analyze students’ background knowledge, the knowledge of effective explanations and demonstrations, together with the knowledge of areas of student difficulty, is pedagogical knowledge. Mathematical pedagogical content knowledge is where mathematical content knowledge and pedagogical knowledge merge.

Typical elementary school mathematics content classes do not address pedagogical content knowledge. Neither do most elementary school mathematics methods classes. It is an area of knowledge which both sides assume comes together in the actual classroom, and which, in fact, seldom does (Ball, 2000). Without mathematical pedagogical content knowledge, elementary teachers can only focus on the “doing” of mathematics and are not able to explain why we “do” it that way. Without mathematical pedagogical content knowledge, elementary teachers present a mathematical topic in only one way — the way they understand it. They are not able to deal with the situation when their students do not understand their way.

It would seem that without mathematical pedagogical content knowledge, neither those elementary teachers who have the strongest math-

ematics content background, nor those experts in pedagogical knowledge and skills, are thoroughly prepared to effectively teach elementary students the mathematics they need to learn. The difference between an elementary school teacher whose students are successful in learning mathematics and an elementary school teacher whose students are not successful may be the teacher's mathematical pedagogical content knowledge. More research needs to be conducted, in how mathematical pedagogical content knowledge is developed and what its relationship is to student learning. Mathematical pedagogical content knowledge holds promise of being the missing keystone of the knowledge base needed to become an effective elementary school mathematics teacher.

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